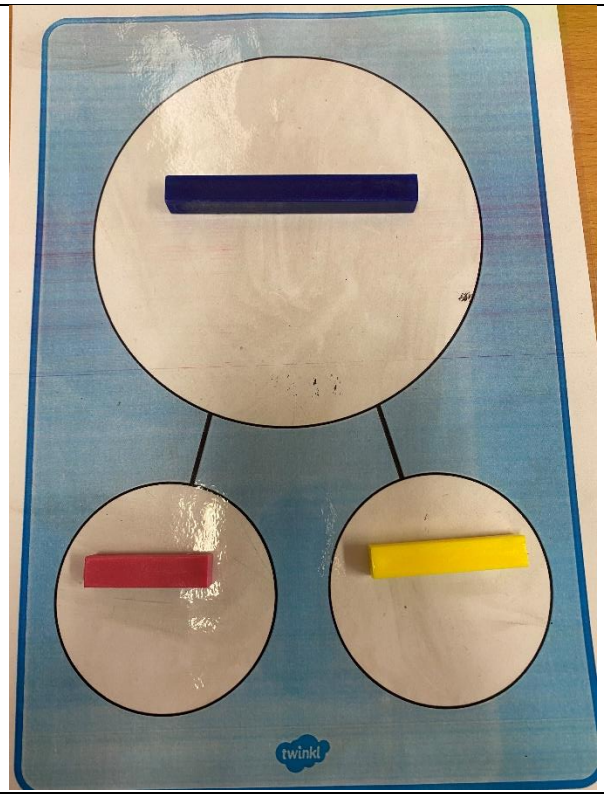
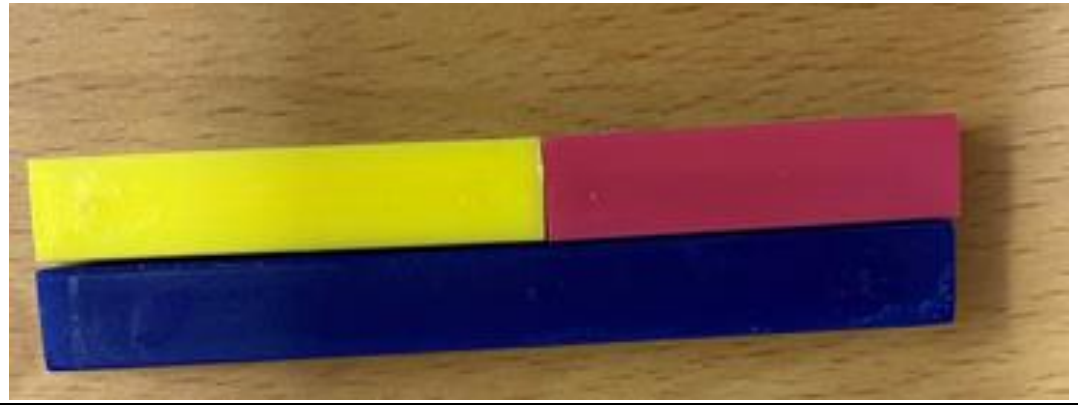
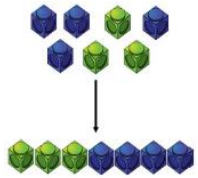
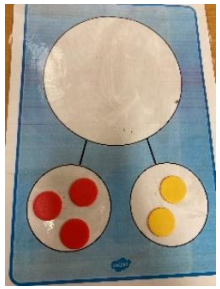
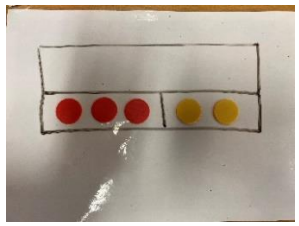
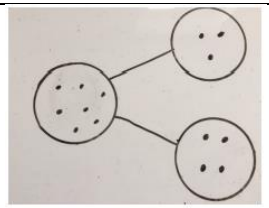
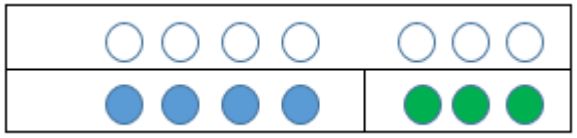
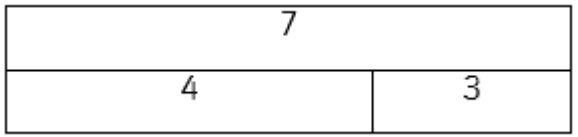
Addition – No Numbers

Main Year Groups	Children should build their vocabulary with Cuisenaire Rods. The part-whole method and the bar model should be introduced with Cuisenaire rods. The rods should not be given numerical values. NB – check for signs of colour deficiency/blindness.
R	 

Addition

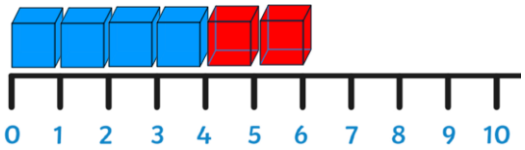
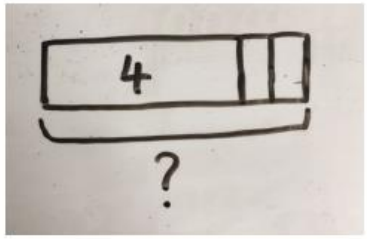
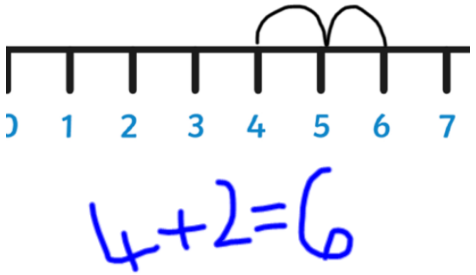
Combining sets of objects

Pre-requisites and explicit pre-practice:
1-1 Correspondence
Counting of objects
Dividing objects into two groups
Understand that the last number counted is the amount in the set.

Main Year Groups	Concrete	Pictorial	Abstract
R, 1, 2	 Combine a variety of objects to make a whole.   *Most children should do this step by the end of R.*	 Children use their own efficient markings to represent the concrete objects.  *Most children should do this abstract step by the end of Y1*	4+3=7  *Most children should do this abstract step by the end of Y1*

Holding a number and counting on.

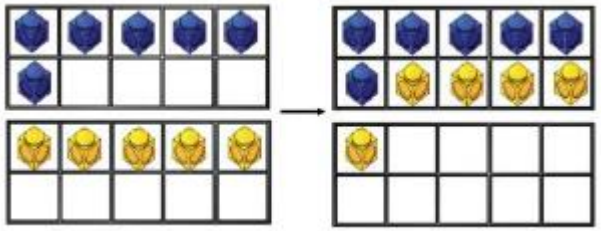
Pre-requisites and explicit pre-practice:
Familiarisation with the number line, looking at patterns, recognising numbers etc.
Oral practice at counting on from a given number
Number recognition
'Holding' a number and understanding that you don't need to start from zero each time.

R, 1, 2	 Children use practical objects with a number line. A bead string will also help.  *Most children should do this by the end of R,*	 A bar model that encourages children to count on rather than count all. *Most children should do this by the end of Y1*	 Abstract number line. *What is 2 more than 4?* *What is 4 plus 2?* *What is the sum of 4 and 2?* *Most children should do this with numbers <20 and with counting back a few steps by the end of R* *Most children should do this by the end of Y1*

Partition and bridge

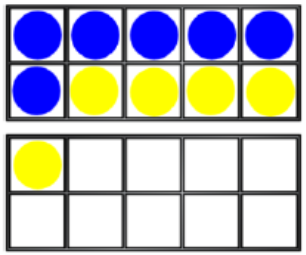
Pre-requisites and explicit pre-practice:
Instant recognition of amounts on a tens frame
Understanding of part-whole
Recall of bonds to 10 –VITAL!
Understanding and fluency with teens numbers as 10 and 'some' more.
Conversations and exploration about why certain partitions are needed. E. G In the calculation below (6 + 5 = 11), why would partitioning the 5 into a 3 and a 2 not be helpful? This needs to be done LOTS!

1, 2, 3

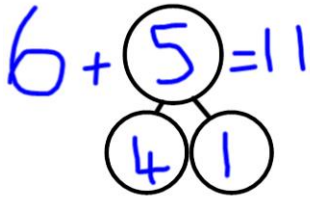


Children begin to partition and bridge by using the tens frames

Most children should do this mentally by the end of Y1. It is vital that children know their bonds to 10 before they can master this.



Children use their own marks to draw in a tens frames. They also then sketch their own tens frames.

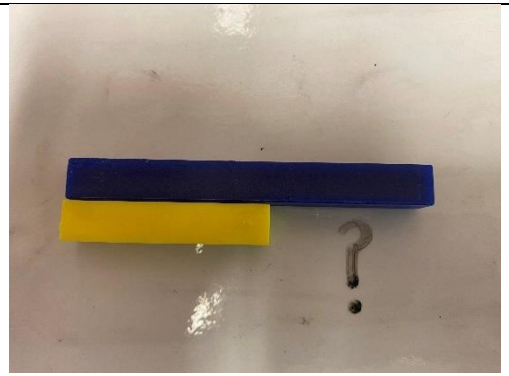
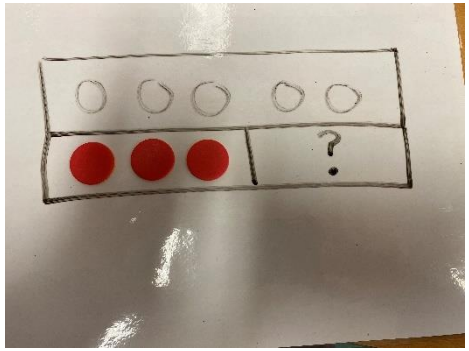
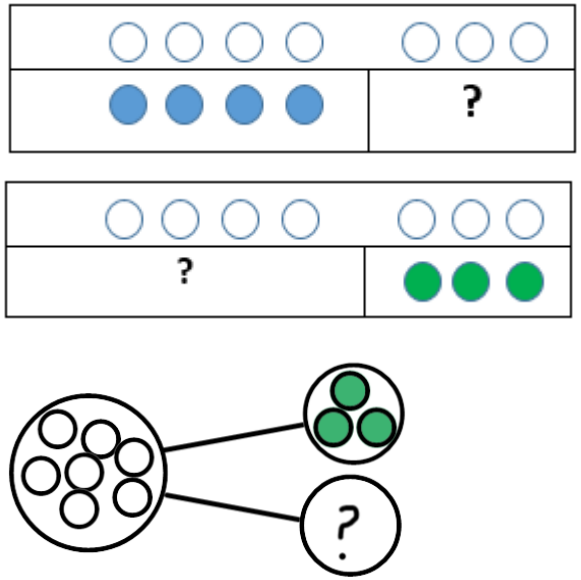
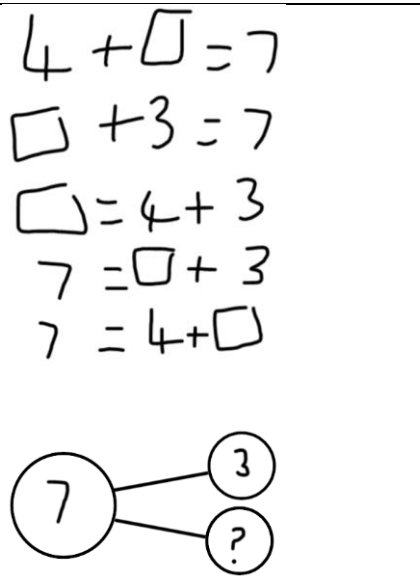


11		
6	5	
6	4	1

Children make jottings onto abstract calculations before paritioning and bridgeing mentally.

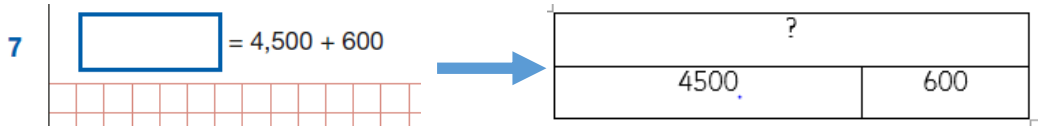
Most children should do this with written scaffold by the end of Y2
Most children should do this mentally by the end of Y3

Use of part-whole and bar model in KS1 for 'missing number' problems.

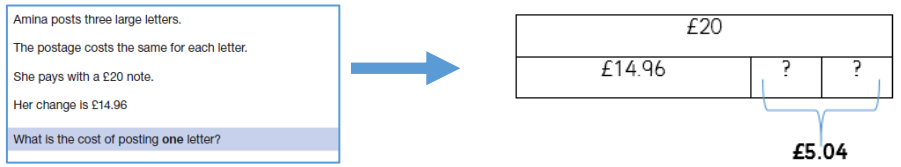
Main Year Groups	Concrete	Pictorial	Abstract
1, 2	 		

Most pupils should be able to solve missing number problems by the end of Y1

Children should continue to use the bar model (or part whole) through to Y6 to solve missing number problems in a variety of situaitons:



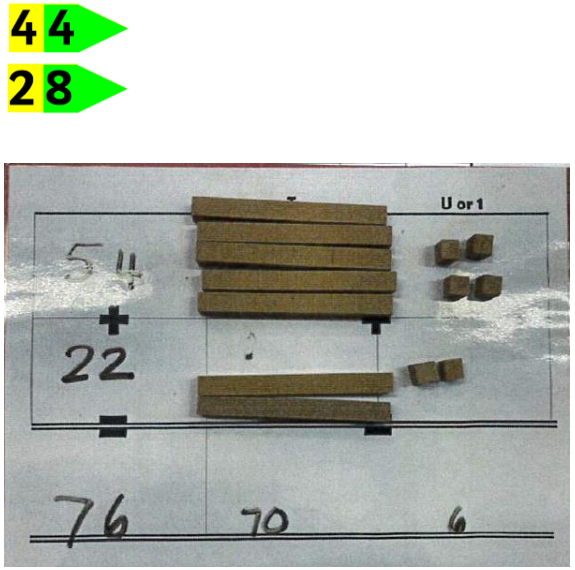
or



Adding O to TO and TO to TO (No crossing boundaries)

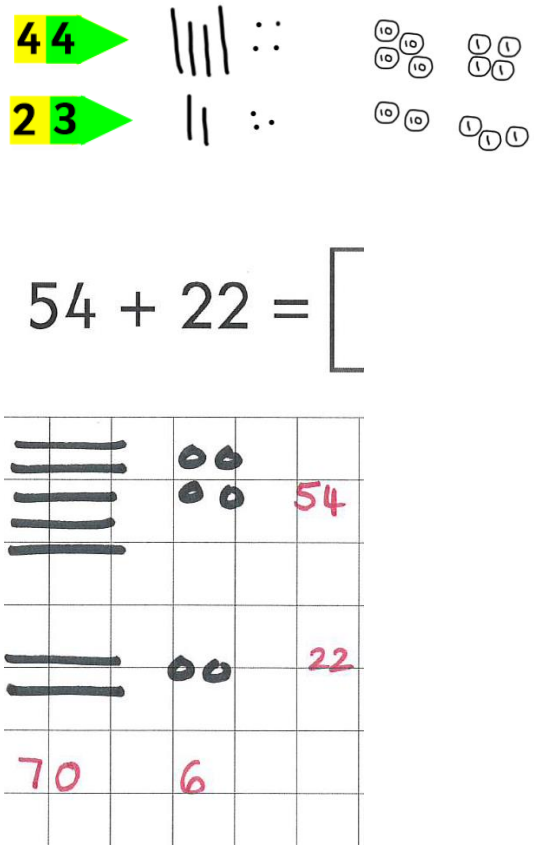
Pre-requisites and explicit pre-practice:
Counting in tens
Mental addition of multiples of ten
Mental addition/recall of adding one-digit numbers to one-digit numbers
Partitioning numbers into tens and ones
Representing numbers with arrow cards, dienes and place value counters.

2,3

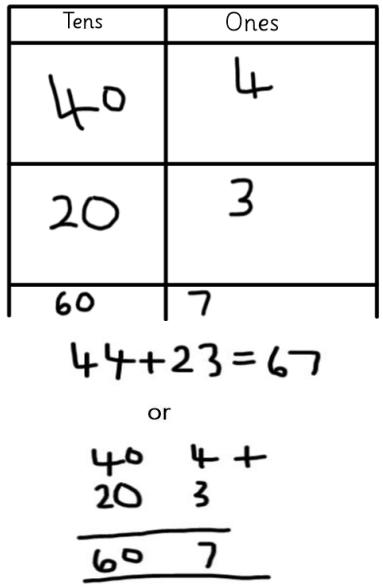


44 + 28 =
Children to use Dienes to add two 2-digit numbers by combining tens and ones. Place value arrow cards can be used for partitioning.

Most children should do this by the end of Y2



Children represent their calcualtions with drawings. They begin to organise them vertically.
Most children should do this by the end of Y2

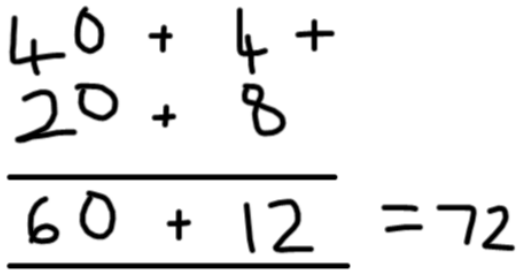
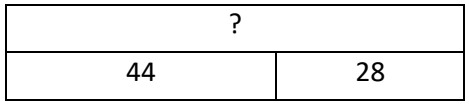


?	
44	23

Children represent their calcualtions horizontally and vertically.
Most children should do this abstract step by the end of Y3. (This can be with the use of the place value frame as a scaffold.)

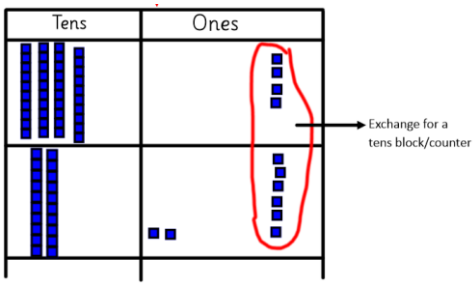
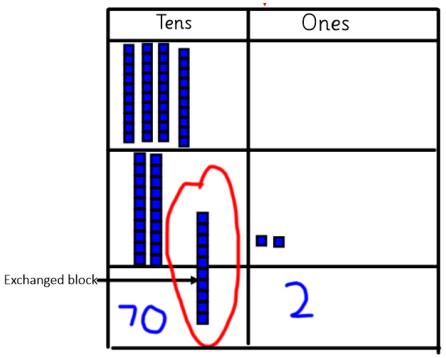
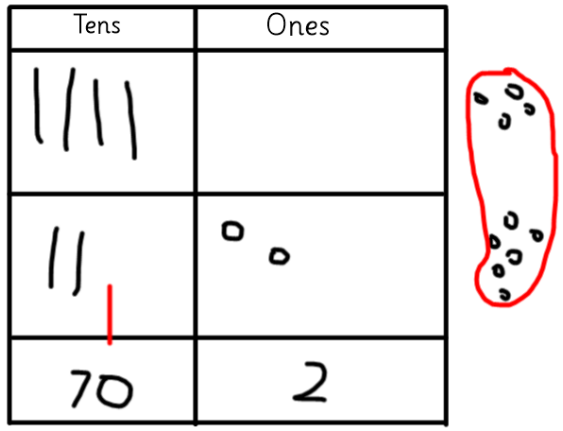
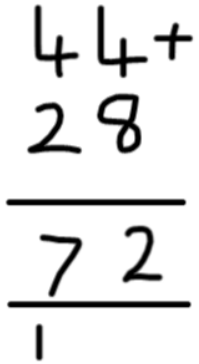
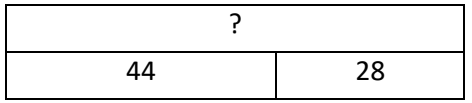
Adding TO to TO (Crossing boundaries – step 1)

Pre requisites and explicit pre-practice:
Exchanging ones for tens with Dienes and place value counters
Understand that ten ones are the same as one ten.
Mentally adding a number between 11 and 19 inclusive to a multiple of 10.

3,4	As above 44 + 28 = Children to use Dienes and place value counters to add two 2-digit numbers by combining tens and ones. Place value arrow cards can be used for partitioning.	As above Children represent their calcualtions with drawings. They organise them vertically.	  Children represent their calcualtions horizontally and vertically. *Most children should do this abstract step by the end of Y3. (This can be with the use of the frame as a scaffold.)*

Adding TO to TO (Crossing boundaries – step 2)

Pre-requisites and explicit pre-practice:
Exchanging ones for tens with Dienes and place value counters
Understand that ten ones are the same as one ten.
Mentally adding a number between 11 and 19 inclusive to a multiple of 10.

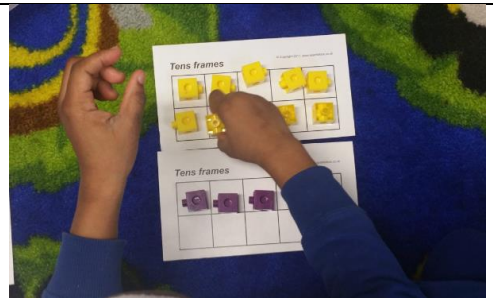
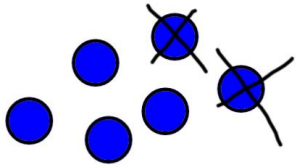
3,4	Using the frames as for no boindaries.   44 + 28 = Children to use Dienes and place value counters to add two 2-digit numbers by combining tens and ones. They exchange the counters/blocks when the ones column has more than 9 in total. *Most children should do this abstract step by the end of Y3. (This can be with the use of the frame as a scaffold.)*	 Children represent their calcualtions with drawings. They organise them vertically. *Most children should do this abstract step by the end of Y3. (This can be with the use of the frame as a scaffold.)*	  Children represent their calcualtions vertically. The bar model will help develop understadning of inverse operations. *Most children should do this abstract step by the end of Y4. (This can be with the use of the frame as a scaffold.)*

Subtraction

Subtracting Ones

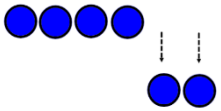
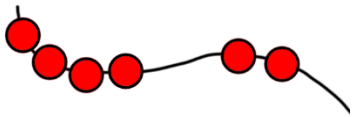
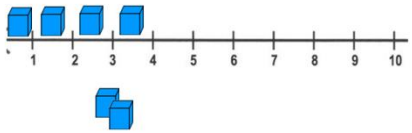
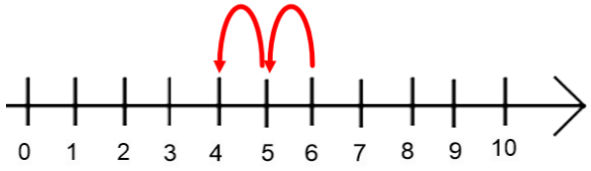
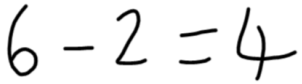
Pre-requisites and explicit pre-practice:

1-1 Correspondence
Counting of objects
Number names
Understanding of more/fewer
Understand that the last number counted is the amount in a set

Main Year Groups	Concrete	Pictorial	Abstract
R, 1, 2	 *Most children should do this by the end of R.*	 (6 – 2 = 4) *Most children should do this by the end of R.*	6 – 2 = 4 *Most children should do this by the end of Y1*

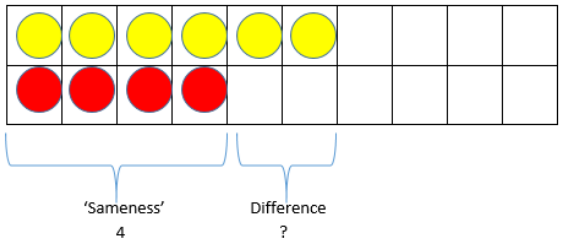
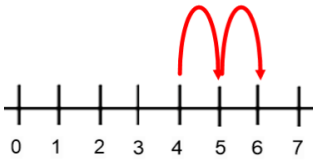
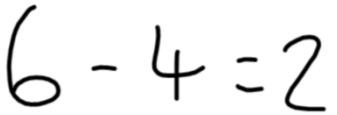
Counting Back

Pre-requisites and explicit pre-practice:
Number names
Count backwards from 10
1-1 correspondence
(Children should continue to use counting back when introduced to fractions, decimals and negative numbers in years 4-6)

R, 1, 2	 Child knows how many to start with. As they remove each object, they count backwards. "6, 5, 4…)  Bead string can be used when counting backwards.  Objects can be placed on a number line and then taken away.		*"Put six in your head and count back two. What number are you on?"*  *Most children should do this with numbers <20 and with counting back a few steps by the end of R* *Most children should do this by the end of Y1*

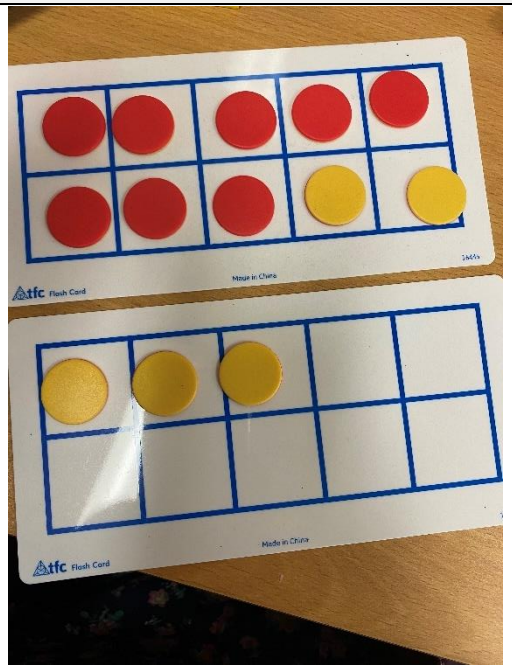
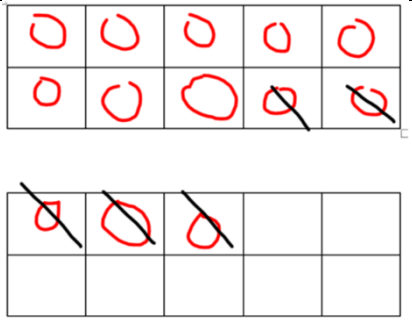
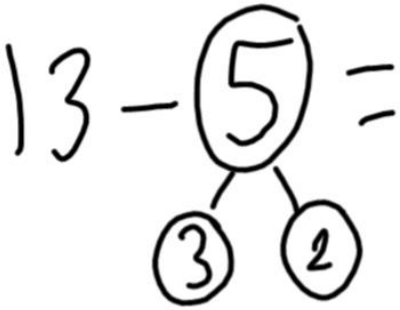
Finding the difference

Pre-requisites and explicit pre-practice:
Number names
Count backwards from 10
1-1 correspondence
(Children should continue to use counting back when introduced to fractions, decimals and negative numbers in years 4-6)

1, 2, 3	*Most children should be able to describe the difference between two towers of cubes by the end of R.* *Objects can be laid on a bar model.*  	 (What is the difference between 4 and 6? 6 – 4 = ?) *Most children should be able to do this by the end of Y1.*	*"Art has 4 sweets and Bart has 6 sweets. How many more does Bart have?"*  **IMPORTANT* Children should be taught that finding the difference is not always an efficient strategy. For example, 103 – 4 = ? It would be more efficient to count back)* *Most children should be able to do this by the end of Y1.*

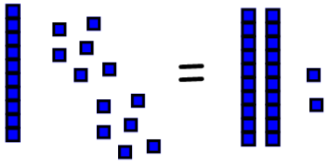
Partition and Bridge

Pre-requisites and explicit pre-practice:
Number names
Count backwards from 10
1-1 correspondence
(Children should continue to use counting back when introduced to fractions, decimals and negative numbers in years 4-6)

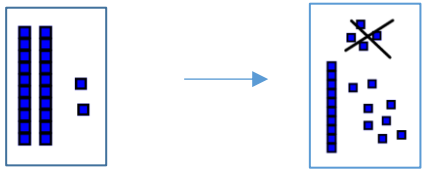
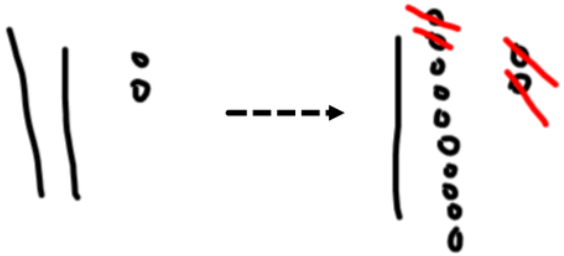
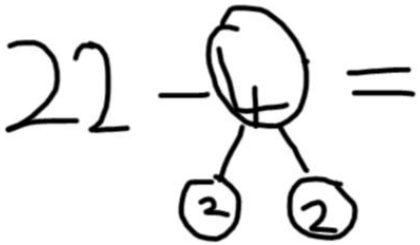
1, 2, 3	 *Most children should do this by the end of Y1*	 *(13 – 5 = ?)* *Subtract 3* *Subtract 2* *Most children should do this by the end of Y1*	 *Most children should do this by the end of Y2* *Most children should do this mentally by the end of Y3*

Regroup Tens and Ones

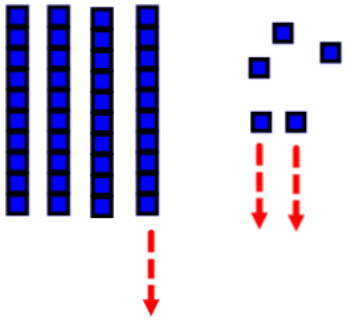
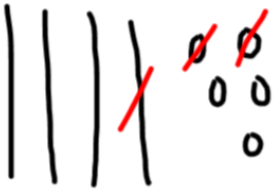
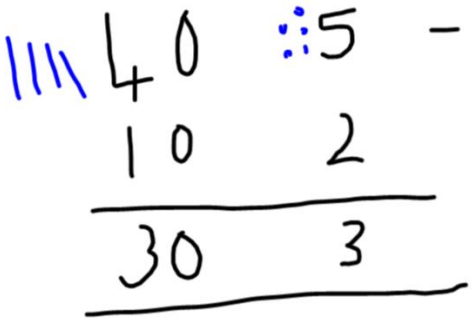
Pre-requisites and explicit pre-practice:
Number names
Count backwards from 10
1-1 correspondence
Swapping/exchanging ones blocks for tens blocks

<table>
<tr>
<td>2, 3</td>
<td>

Before calculating, children need to practise JUST exchanging.

Most children should be able to do simple exchange without calculation, by the end of Y1.
Then…
22 – 4
(One of the tens needs to be exchanged for ones)


Most children should do this mentally by the end of Y2
</td>
<td>
22 – 4 =

<table>
<tr><td colspan="2">22</td></tr>
<tr><td>4</td><td>18</td></tr>
</table>
Most children should do this mentally by the end of Y2
</td>
<td>

Most children should do this by the end of Y2
<table>
<tr><td colspan="2">22</td></tr>
<tr><td>4</td><td>18</td></tr>
</table>
Most children should do this mentally by the end of Y2
</td>
</tr>
</table>

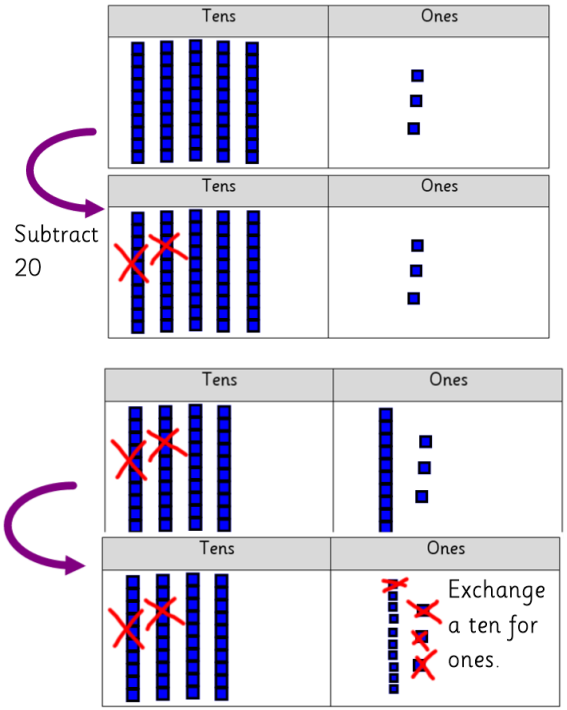
Column Subtraction (No Regrouping)

2, 3	45 – 12 =  Dienes should be placed onto place value grids to emphasis their value. *Most children should do this by the end of Y2*	45 – 12 =  **45** **12** | **33** *Most children should do this by the end of Y2*	 **45** **12** | **33** *Most children should do this by the end of Y3*

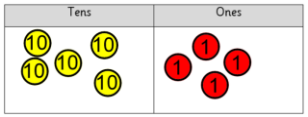
Column Subtraction (With Regrouping) – Step 1

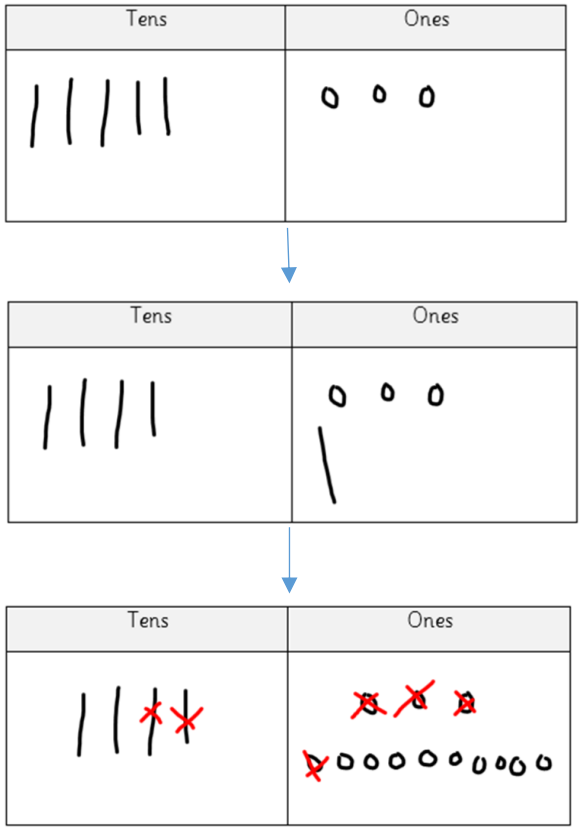
3

53 - 24 =

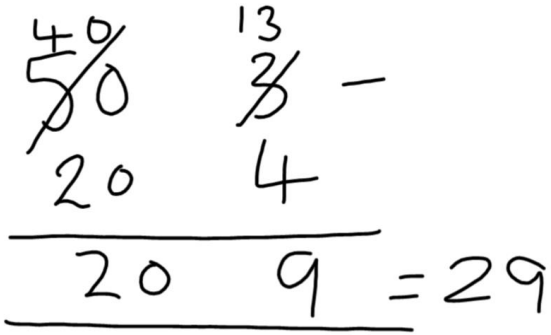


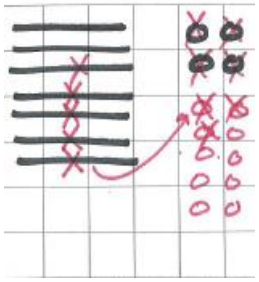
Place Value Counters should also be used when children are secure with place value.

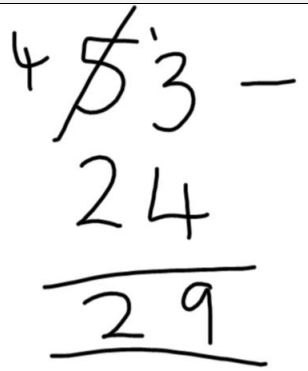




Most children should do this by the end of Y3



	Most children should do this by the end of Y2	74 − 47 =  *Most children should do this by the end of Y2*	*Most children should do this by the end of Y3*

Column Subtraction (With Regrouping) – Step 2			
3,4	As above	As above	 *Most children should do this by the end of Y4* *Y5 and Y6 children continue to use this method for integers and decimal fractions.*

Multiplication

Doubling

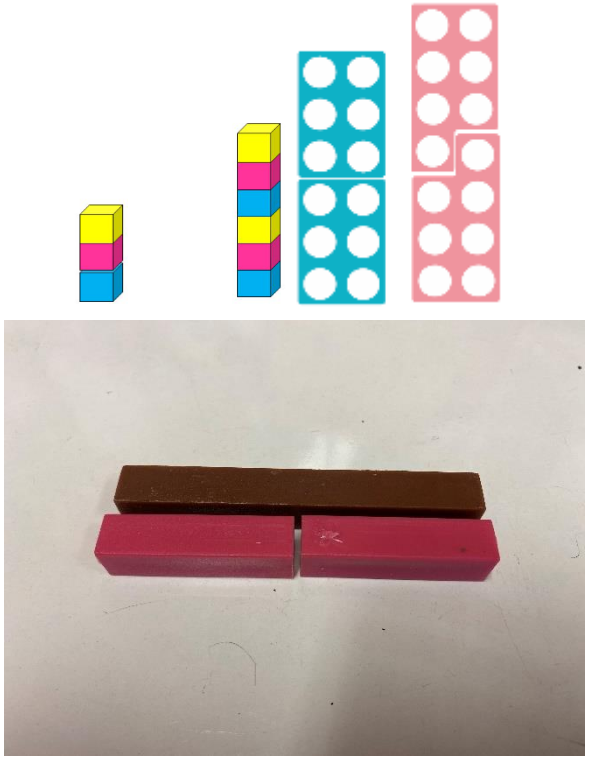
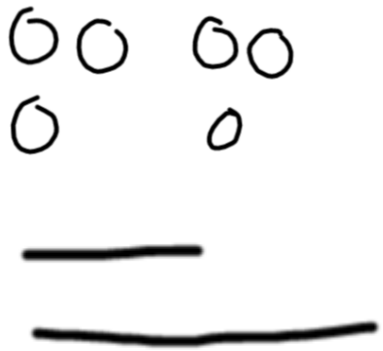
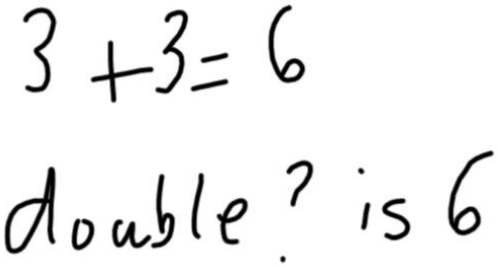
Pre-requisites and explicit pre-practice:

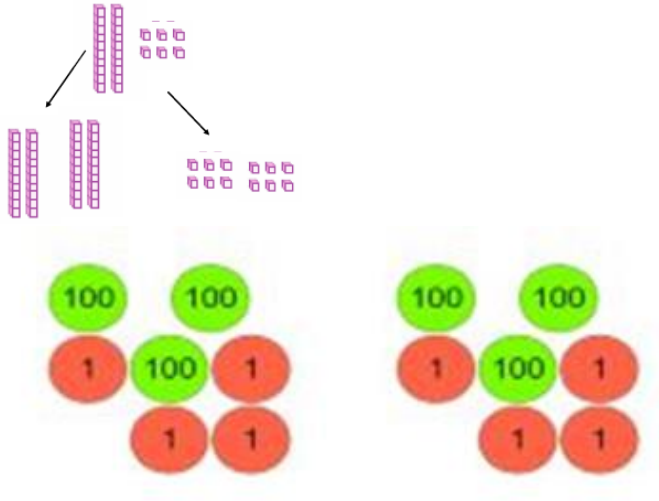
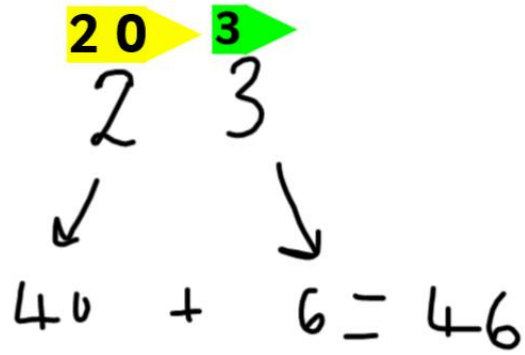
1-1 Correspondence

Counting of objects

Dividing objects into two equal groups

Understand that the last number counted is the amount in the set.

Main Year Groups	Concrete	Pictorial	Abstract
R, 1		 *Most children should do this by the end of Y1*	 *Most children should do this by the end of Y1*

	Children understand the concept of doubling by taking an amount and making it again. They should be able to explain when resources show a 'not double' and begin to explain how to make it a double. *Most children should do this by the end of R (Children are not expected to recite doubles)*		
2,3	 *Most children should be able to do this (to the relative numbers) by the end of Y2.*	 *Most children should be able to do this (to the relative numbers) by the end of Y2.*	 *Most children should be able to do this (to the relative numbers) by the end of Y3 (including crossing boundaries, e.g. double 27)*

Multiples, Equal Groups, Repeated Addition

Pre-requisites and explicit pre-practice:

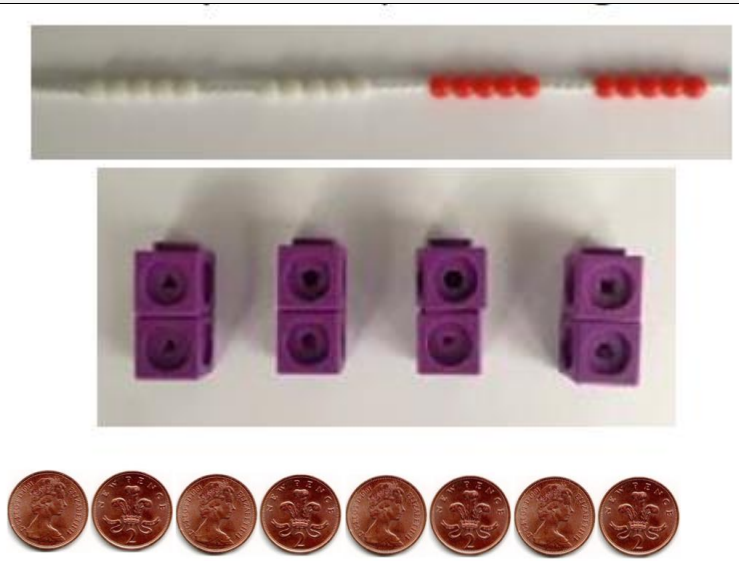
1-1 Correspondence

Counting of objects

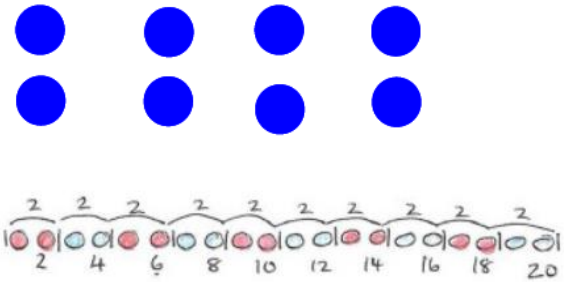
Dividing objects into two equal groups

Understand that the last number counted is the amount in the set.

R, 1



Most children should do this by the end of 1 counting in 2s, 5s and 10s.



Most children should do this by the end of Y1

2, 4, 6, 8, 10

5, 10, 15, 20, 25, 30

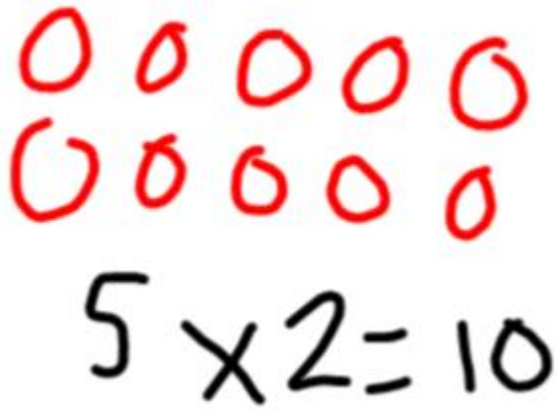
2 + 2 + 2 + 2 + 2 = 10

10				
2	2	2	2	2

Most children should do this by the end of Y1

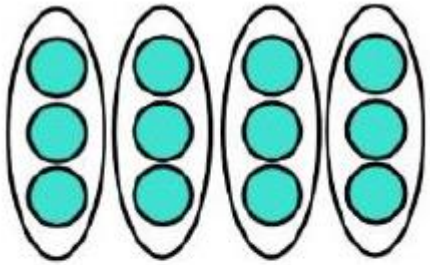
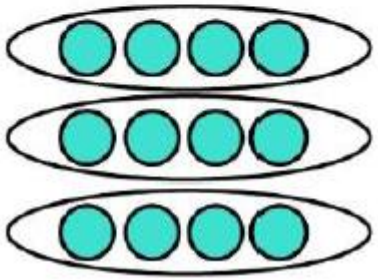
Understanding Arrays

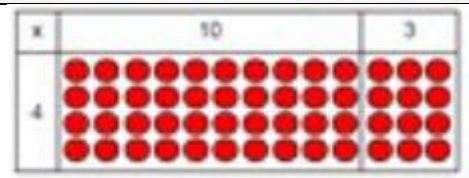
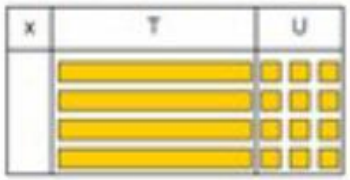
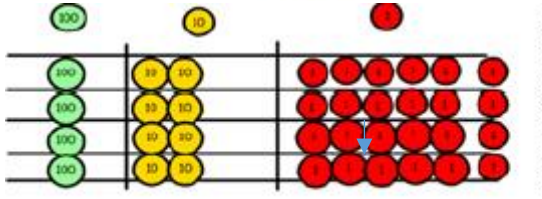
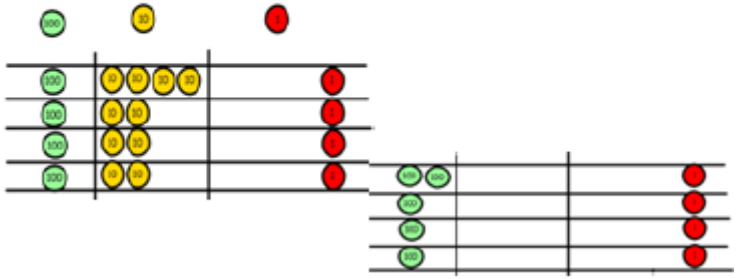
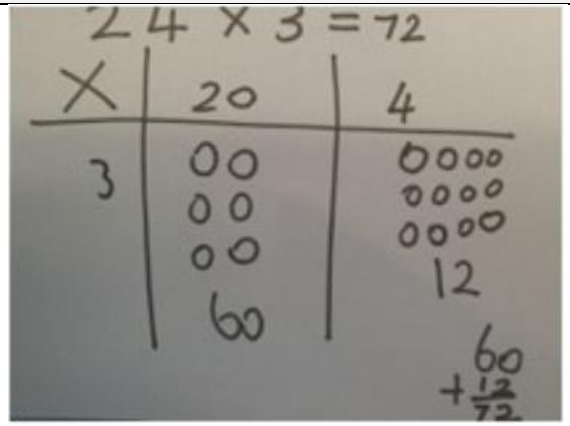
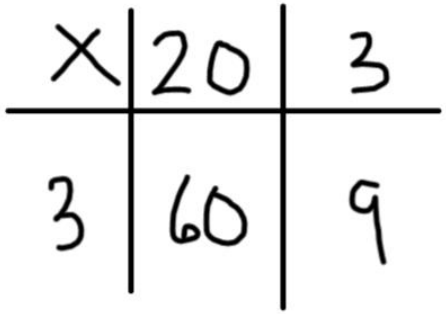
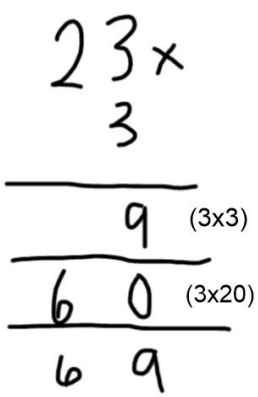
Pre-requisites and explicit pre-practice:
Spatial understanding of equal rows and columns

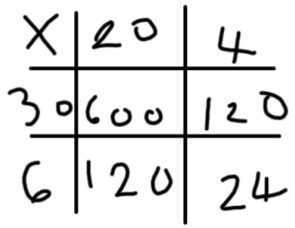
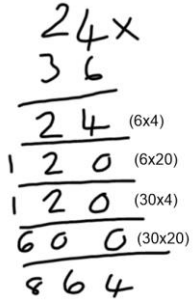
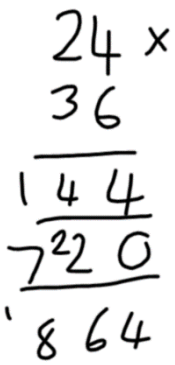
1,2,3	  *Most children should do this practically by the end of R*	 *Most children should do this by the end of Y1*	 *Most children should do this by the end of Y2*

Commutativity

Pre-requisites and explicit pre-practice:
Understanding of x and =
Understanding of the array

2,3	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.   *Most children should do this practically by the end of Y2*	  *Most children should do this by the end of Y2*	 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15 *Most children should do this by the end of 2*

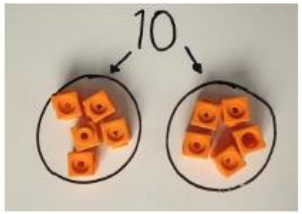
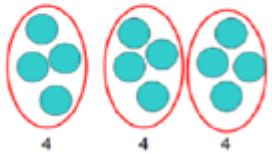
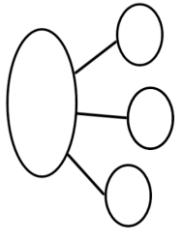
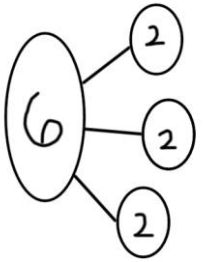
Grid Method			
3,4	   Add up each column, starting with the ones making any exchanges needed  *Most children should be able to do this by the end of Y3.*	 *Most children should be able to do this by the end of Y3.*	 *Children should be able to do this by the end of Y3.*  *Most children should be able to do this by the end of Y4. When children are secure, they should progress on to HTO x O*

Column Multiplication TO x TO			
5, 6			 *Children should be able to do this by the end of Y4.*   *Most children should be able to do this by the end of Y5.*

Division

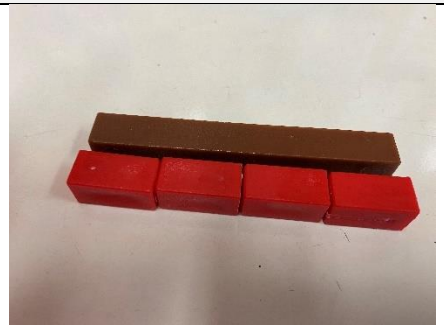
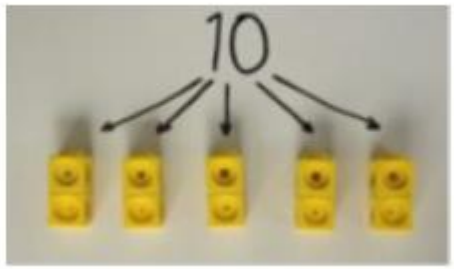
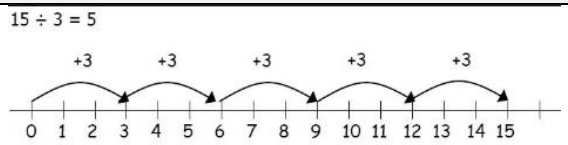
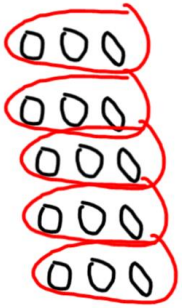
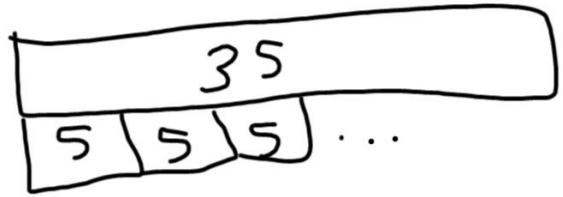
Sharing

Pre-requisites and explicit pre-practice:
Understanding of sharing equally
Understanding of repeated addition

Main Year Groups	Concrete	Pictorial	Abstract
R, 1, 2, 3	  	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 Sharing:  12 shared between 3 is 4 	No use of the division sign in R or Y1. Children might write sentences: *6 sweets shared between 3 people means they each get 2 sweets.*  6 2 2 2 *By the end of Y2, most children should use and understand the division sign: 6 ÷3 = 2*

Grouping

Pre-requisites and explicit pre-practice:
Understanding of equal groups
Skip counting for relevant multiples
Understanding of repeated addition

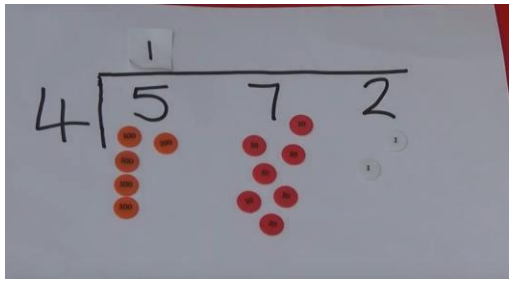
R, 1, 2, 3	  Objects should be grouped into equally sized groups in problem solving contexts and real-life contexts.  The bead string can be used for grouping. *Most children will be able to do this by the end of Y1*	  Children build arrays by grouping into threes here. 15 ÷ 3 = *How many threes are in 15?* *15 grouped into threes.* *Most children will be able to do this by the end of Y2.*	Children begin to count in step sizes to work out the answers to division questions. 35 ÷ 5 = 7 *5, 10, 15, 20, 25, 30, 35*  Children use the bar model to support counting in step sizes. *Most children will be able to do this by the end of Y2.*

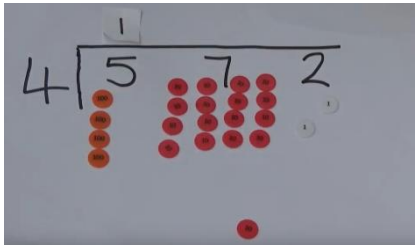
Short Division

Pre-requisites and explicit pre-practice:
Secure times tables and related division facts
Understanding of remainders
Understanding of value of place-value counters

4,5,6

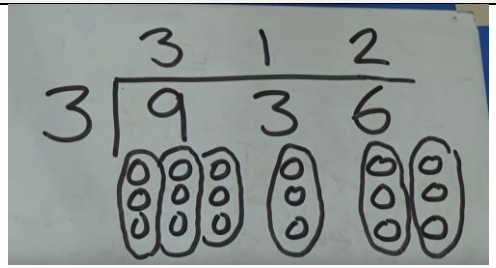
With exchanging



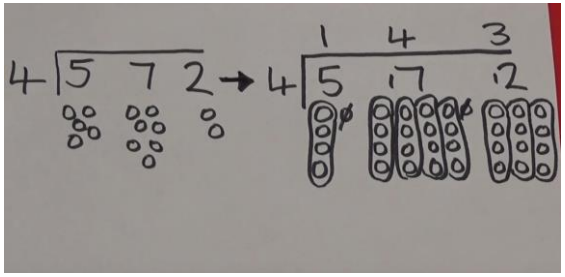


The left over 100 counter needs to be exchanged for ten 10 counters under the 7. And so on…

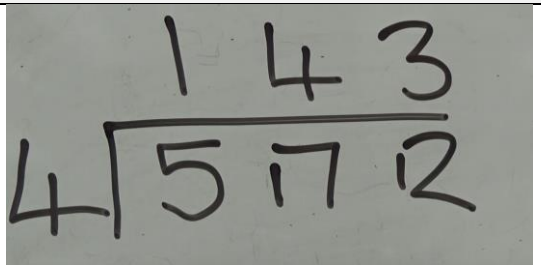
Most children should be able to do this by the end of Y4



With exchanging



Most children should be able to do this by the end of Y4



Most children should be able to do this by the end of Y4

Long Division

Pre-requisites and explicit pre-practice:
Mental addition of 2-digit numbers
Understanding of equal groups
Understanding of remainders

6